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# Weaving with Ropes to Parse Mathematical Abstraction

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**Abstract:** We draw on constructionism to design a group rope weaving activity as an illustration of concepts in matrix algebra. The study, conducted in an undergraduate Informatics class, involved a rope-weaving activity to assess the concept of cloth separability and the matrix representation of weaving patterns. Results showed significant learning gains, with distinct approaches observed: one group emphasizing physical manipulation, the other relying on mathematical principles. This study underscores the value of tangible engagement in understanding abstract concepts.

## Introduction and background

Matrix algebra is especially prominent in STEM contexts and a steppingstone for more advanced mathematics. With its inherent ties to computation and binary nature (Harlizius-Klück, 2017), weaving is a good context to situate a tangible engagement with matrix algebra, which holds the promise for more effective learning of abstract ideas that students tend to find difficult to grasp. Beyond its binary nature, weaving has meaningful connections across areas in advanced mathematics such as set theory (relating to cloth separability) and vector calculus (relating to the quality of the cloth). This poster presents a cross-case comparison of two groups engaged in a rope weaving activity as an illustration of applied concepts in matrix algebra in the context of textiles. We asked*: How did the activity impact student understanding of the matrix representation of weaving patterns and the associated concept of cloth separability?*

Weaving's binary nature lies in the ability to encode patterns in a matrix, with 0s and 1s, representing threads as row or column vectors (see Figures 1a-c). Such binary representation further allows exploration of cloth separability, or whether a cloth is going to hold together as a single cloth (Speer et al., 2023). In the case that there are groups of threads, or sets, that do not interlace, the cloth will "fall apart" (see Figures 1c where A and B are examples of two sets that do not interlace). To design an activity examining these concepts, we ground this poster in constructionism (Papert, 1980), which positions learners as constructors of their own knowledge in interaction with "objects-to-think-with," which help create and solidify mental models. In the constructionist tradition, tinkering is a natural process that supports learning and lends itself to environments with low floors and high ceilings, challenging and stimulating for all learners (Resnick & Silverman, 2005).

## Methods

This exploratory pilot study is set in the context of an elective undergraduate Informatics class in Southern California, intersecting notions in robotics, matrix algebra, and textile arts. To better understand the matrix representation of weaving patterns and the concept of cloth separability, we designed a hands-on rope-weaving activity, in which students re-created 3 patterns and worked in groups of 3-4 ($N = 19$) to determine if these patterns hold together or separate. We aimed to form groups with diverse expertise, as determined by pre-course surveys. We developed a codebook to score a pre- and post- assessment ($n = 5$ items, focused on cloth separability in the matrix context) on a 0-2 scale for a 10-point total, establishing IRR with 20% of the data first. We collected video data with 360-degree cameras (25 minutes per group), set on the floor to capture conversation and the weaving process, and individual reflection data to gauge students' approach. Quantitatively, we conducted a t-test to evaluate student learning gains (post-pre). Qualitatively, we present a cross-case analysis of two groups (Stake, 1995), who showed among the largest learning gains, but selected vastly different strategies - one reliant on the physicality of the ropes (G1), and the other – entirely on the matrix algebra behind it (G2).

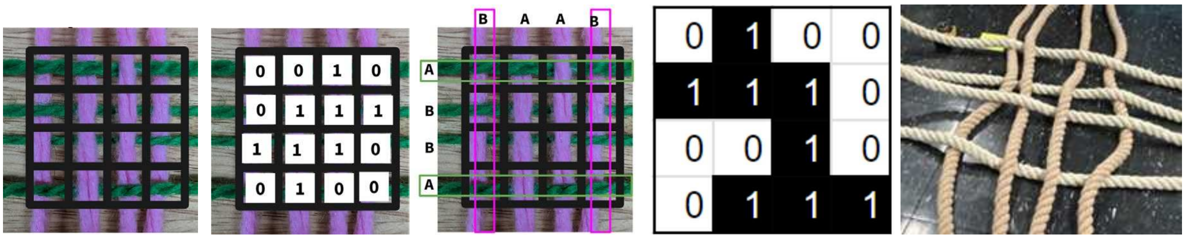
## Findings

We conducted a paired-sample t-test and found that, on average, students increased their scores by 3.2 points, which was statistically significant ($p < .001$). Both focal groups showed learning gains above average, with students in Group 1 showing one of the largest learning gains (a 4.5-point increase in scores, on average).

The physical manipulation of the ropes ignited a spark of understanding for students in Group 1, as they worked collaboratively to weave the 3 patterns, making sense of the matrix representation (see Figures 1d and 1e). Upon weaving the first pattern and lifting one set of ropes off the other, one student said, “That’s actually so cool.” The visual of one set of ropes separating from another illuminated in meaningful ways the core idea of separability, grounding it in the physical engagement with the ropes and providing a scaffold toward a deeper understanding of the encoded abstractions. Reflecting on the activity, another student said, “A large part of our strategy was due to having the ability to manipulate the ropes physically, and it really helped visualize the problem.” Indeed, during the activity, students would tug at the ropes to gauge whether they were connected to other ones. As such, the ropes acted as objects-to-think-with for students, letting them tinker and experiment.

On the other hand, Group 2’s strategy was strictly reliant on the underlying mathematical principles, with one student, a Math major, taking the lead in the activity. In his reflection, he said, “I had a very good understanding of matrix math, so I was able to guide my group in calculating whether or not the ropes would fall apart.” Even so, physically engaging the ropes, he said, helped demonstrate to his peers the underlying mathematical ideas and what to look for when determining which strands belong in the same set, in line with our conclusion that physical engagement is a good scaffold toward a deeper understanding of concepts. But as part of their approach, Group 2 primarily focused on the binary notation to figure out which ropes belong in the same set, testing out their hypothesis thereafter to prompt the conclusion, “It falls apart, we just proved it.”

**Figures 1a-e**
*Binary Notation of Weaving (1a-b) with 1: Purple over Green and 0: Green over Purple, and Sets that Do Not Interlace (1c). Group 1 Weaving a Binary Pattern (1d) with Ropes (1e)*




(1a) (1b) (1c) (1d) (1e)

## Toward design of tangible engagement with abstract ideas

Constructionism, which emphasizes the importance of learning by doing, provides a useful lens to analyze how the collaborative weaving process can serve as a representation of the binary nature of weaving. The tangible and experiential aspects of the activity created a rich learning environment for students to actively construct their knowledge around the matrix representation of weaving patterns and improve their understanding. Students' level of comfort with abstraction and math experience determined to what extent they relied on the ropes’ physicality, with the material engagement providing a good scaffold for learners, parsing mathematical abstraction regarding the concept of cloth separability. Observing the two approaches affirms the importance of designing activities with low floors and high ceilings, letting learners select their own approach.

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